Background on EXH

You will find in the literature at least two different definitions of EXH.

EXH number 1
The one generating a contradiction: (Chierchia 2013)

(1) $[\![EXH_{Alt}\ \varphi]\!]^{g,w0}$=T iff
$[\![\varphi]\!]^{w0}$=T & $\forall q[q \in \{\lambda w.[\![\gamma]\!]^{w} : \gamma \in Alt\}\ \&\ \lambda w'.[\![\varphi]\!]^{w'} \not\models q \rightarrow \neg q(w)]$

This EXH:

- Asserts the prejacent;
- Negates all alternatives that are not entailed ($\not\models$) by the prejacent

Clarification:

- ALT here is a variable that hosts the alternatives
- Alternatives are propositions
- They can be formed structurally
- They can be lexically specified
- There are different ways of constructing alternatives

EXH number 2

The one based on innocent exclusion (refusing to negate the alternatives if the resulting combination would lead to a contradiction with the prejacent): (Fox 2007)

(2) $[\![EXH_{ALT}\ \ \varphi\ ]\!]^{g,w0} = [\![\varphi]\!]^{\ g,w0}\ \&\ \ \forall q\ [q \in IE(\lambda w'.[\![\varphi\ ]\!]^{g,w'}, ALT) \rightarrow \neg q(w)]$

(3) IE(p, ALT) = $\cap$\{ALT' $\subseteq$ALT: ALT' is a maximal subset of ALT s.t. $ALT'^{\neg} \cup \{p\}$ is consistent\}
$ALT'^{\neg} = \{\neg p' : p' \in ALT'\}$

This EXH:

- asserts the prejacent
- negates all alternatives that are in every maximal set of alternatives that can be negated together with the assertion of the prejacent
- it looks ahead a little bit and will not negate the alternatives if their combination will lead to a contradiction

They are very similar, but not the same and produce sometimes different results.
Both are used and seem to be necessary.

1. Contradiction based EXH

The first EXH is like 'only': it checks if the alternative is entailed by the prejacent. If it is not entailed, it is negated.

(4) Only [Mary and John]$_F$ came.
(5) The alternatives:
Mary and John came (the prejacent is always an alternative)
Mary came
John came
Bill came
Mark came

Only asserts the sentence that comes before it and negates the alternatives that are not entailed by it.

(6) The negated alternatives:
Bill did not come
Mark did not come
Oleg did not come

We, of course, do not want to negate this alternative: **Mary came,** and this alternative: **John came**, and the prejacent itself

Well, *only* first checks if the alternative is entailed.
Since those are entailed, they are not negated.
All is good.
No contradiction is generated here.

This EXH is used to derive the fact that NPIs need DE environment.

(5) I did not read any book
(6) *I read any book

The respective LFs
(7) [EXH_{ALT} I did not read any book]
(8) [EXH_{ALT} I read any book]

Any is an existential.
In the first case with negation the prejacent gets this meaning:

(9) ⟦I did not read any book⟧= T iff $\neg\exists x$[x is a book & I read x]

Any triggers subdomain alternatives:
Let's assume the books are {a,b,c}

Then, my prejacent can be equivalently represented as follows:

(5) $[\![\text{I did not read any book}]\!] = T$ iff $\neg\exists x[x \in \{a,b,c\} \& \text{I read } x]$

Then, the subdomain alternatives are:

(6)
$\neg\exists x[x \in \{a,b\} \& \text{ I read } x]$
$\neg\exists x[x \in \{a,c\} \& \text{ I read } x]$
$\neg\exists x[x \in \{b,c\} \& \text{ I read } x]$
$\neg\exists x[x \in \{a\} \& \text{ I read } x]$
$\neg\exists x[x \in \{b\} \& \text{ I read } x]$
$\neg\exists x[x \in \{c\} \& \text{ I read } x]$

- Because all of them are entailed by the original, EXH just has nothing to negate here.
- Because EXH did not do any work here, we correctly capture the fact that the sentence with negation is grammatical.

In the positive sentence, the situation is different:

(7) $[\![\text{I read any book}]\!] = T$ iff $\exists x[x \in \{a,b,c\} \& \text{I read } x]$

Subdomain alternatives:

(8)
$\exists x[x \in \{a,b\} \& \text{ I read } x]$
$\exists x[x \in \{a,c\} \& \text{ I read } x]$
$\exists x[x \in \{b,c\} \& \text{ I read } x]$
$\exists x[x \in \{a\} \& \text{ I read } x]$
$\exists x[x \in \{b\} \& \text{ I read } x]$
$\exists x[x \in \{c\} \& \text{ I read } x]$

None of them individually is entailed by the prejacent. EXH negates them all and we get a contradiction:

(9)
$\exists x[x \in \{a,b,c\} \& \text{I read } x]$ &
$\neg\exists x[x \in \{a,b\} \& \text{ I read } x]$
$\neg\exists x[x \in \{a,c\} \& \text{ I read } x]$
$\neg\exists x[x \in \{b,c\} \& \text{ I read } x]$
$\neg\exists x[x \in \{a\} \& \text{ I read } x]$
$\neg\exists x[x \in \{b\} \& \text{ I read } x]$
$\neg\exists x[x \in \{c\} \& \text{ I read } x]$

Contradiction!

Thus, we derive the pattern.

- In a positive sentence 'any' +EXH would result in a contradiction!
- The idea is that 'any' must be c-commanded by EXH
- Its you choice to place negation in the sentence or not
- If you do not place negation or another DE operator above, you will create a contradiction
- This contradiction arises due to the combination of functional elements 'any' and 'EXH'
- For this reason the sentence is perceived as ungrammatical

2. Why we need EXH based on IE?

Free choice inferences under existential modals: Fox 2007:

Simple disjunction:

(10) Mary ate ice cream or cake. $a \vee b$

Inference: ~Mary didn't eat both

The conjunction of the disjunctive alternatives is *false*: $\neg(a \wedge b)$

Free choice disjunction:

(11) Marry is allowed to eat ice cream or cake. $\Diamond(a \vee b)$

Inference: ~Marry is allowed to eat ice cream and Mary is allowed to eat cake.

The conjunction of the disjunctive alternatives is true: $\Diamond a \wedge \Diamond b$

To derive the latter pattern, we need to assume that the alternatives for $\vee$ do not only include the conjunction as an alternative, but also each individual conjunct.

But then in a simple case without a modal, we do not want the first kind of EXH:

(12) EXH Mary ate ice cream or cake.

(13) ALT: {Mary ate ice cream or cake (the prejacent), Mary ate ice cream, Mary ate cake, Mary ate ice cream and cake}

- The contradiction based EXH creates a problem here:
- 'Mary ate ice cream or cake' does not entail any of the individual disjuncts.
- So that EXH would simply negate them
- In the end, we will get a **contradiction** with the prejacent:
  'Mary ate ice cream or cake, but she did not eat ice cream, she did not eat cake'

The innocent exclusion based EXH does better in this case.

The procedure is this:

- first, we create all maximal sets of alternatives that can be negated together with the assertion of the prejacent;
- then, we pick the alternatives that are in every such set;
- then, we negate them and only them

(14) Max sets of alternative that can be negated together with the prejacent:
{Mary ate ice cream, **Mary ate ice cream and cake** }
{ Mary ate cake, **Mary ate ice cream and cake** }

Only one alternative is in both sets: negate it and conjoin with the prejacent:

(15) Mary ate ice cream or cake and Mary did not eat both.
So far so good!

We can of course place EXH over existential modal:

(16) EXH Marry is allowed to eat ice cream or cake.

(17) Alt:
{ Marry is allowed to eat ice cream or cake
Marry is allowed to eat ice cream,
Marry is allowed to eat cake,
Marry is allowed to eat ice cream and cake}

Nothing is different here from the case above
Again, only one will be in all of the maximal sets: negate it and conjoin with the prejacent:

(18) Marry is allowed to eat ice cream or cake and Marry is not allowed to eat both.

The result is **compatible** with the free-choice inference, but we have not **derived** it yet.

How do we derive this free choice inference?

Solution: double exhaustification:

(19) [EXH_{ALT} [EXH_{ALT} Marry is allowed to eat ice cream or cake]]

(20) Alternatives for the second EXH_{ALT}:
{ EXH_{ALT} Marry is allowed to eat ice cream or cake,

EXH Marry is allowed to eat ice cream,
EXH Marry is allowed to eat cake
EXH Marry is allowed to eat ice cream and cake}

(21) What they actually mean:
{Marry is allowed to eat ice cream or cake and Marry is not allowed to eat both (prejacent),
Marry is allowed to eat ice cream and not allowed to eat cake,
Marry is allowed to eat cake and not allowed to eat ice cream,
Marry is allowed to eat ice cream and cake[1]}

All innocently excludable, we negate all of them other than prejacent:
(22)
Marry is not allowed to eat ice cream or is allowed to eat cake
Marry is not allowed to eat cake or is allowed to eat ice cream
~~Marry is not allowed to eat ice cream and cake~~ (non informative because it repeats a part of the prejacent)

Now we reason about the first two negated alternatives:
- $\neg p \vee q = p \rightarrow q$
- If Mary is allowed to eat ice cream, then she is allowed to eat cake
- If Marry is allowed to eat cake, then she is allowed to eat ice cream
- The prejacent says that **Marry is allowed to eat ice cream or cake**
- These alternatives give us that if she is allowed to eat one, she is allowed to eat the other one.

The final result:
Marry is allowed to eat ice cream or cake and is not allowed to eat both (**prejacent**) and is allowed to eat cake and is allowed to eat ice cream.

3. We cannot use EXH based on IE to derive the NPI pattern

While in the negative case it works the same as the contradiction based EXH: it has nothing to negate, it also negates nothing in the positive case:

(23) [EXH I read any book]

The prejacent:
(24) $[\![\text{I read any book}]\!] = T$ iff $\exists x[x \in \{a,b,c\} \& \text{I read } x]$

Subdomain alternatives: none of them is IE!
(25)
$\exists x[x \in \{a,b\} \& \text{ I read } x]$
$\exists x[x \in \{a,c\} \ \& \text{ I read } x]$

[1] The second EXH here could not negate any of the alternatives as they are all entailed by the prejacent

$\exists x[x \in \{b,c\}\ \&\ \text{I read } x]$
$\exists x[x \in \{a\}\ \&\ \text{I read } x]$
$\exists x[x \in \{b\}\ \&\ \text{I read } x]$
$\exists x[x \in \{c\}\ \&\ \text{I read } x]$

We can negate maximally 3 of alternatives together with the assertion of the prejacent:
$\exists x[x \in \{a,b,c\}\&\ \text{I read } x]\ \&$
$\neg\exists x[x \in \{a,b\}\ \&\ \text{I read } x]\ \&$
$\neg\exists x[x \in \{a\}\ \&\ \text{I read } x]\ \&$
$\neg\exists x[x \in \{b\}\ \&\ \text{I read } x]$

Or this group:
$\exists x[x \in \{a,b,c\}\ \&\ \text{I read } x]\ \&$
$\neg\exists x[x \in \{a,c\}\ \&\ \text{I read } x]\ \&$
$\neg\exists x[x \in \{a\}\ \&\ \text{I read } x]\ \&$
$\neg\exists x[x \in \{c\}\ \&\ \text{I read } x]$

Or this group:
$\exists x[x \in \{a,b,c\}\&\text{I read } x]\ \&$
$\neg\exists x[x \in \{b,c\}\ \&\ \text{I read } x]\ \&$
$\neg\exists x[x \in \{b\}\ \&\ \text{I read } x]\ \&$
$\neg\exists x[x \in \{c\}\ \&\ \text{I read } x]$

- But none of the alternatives is in every group that can be negated together with the assertion of the prejacent;
- None of them is IE and EXH has nothing to do here;
- So no contradiction is generated;
- EXH applies vacuously in the positive and in the negative case alike;
- Thus, we do not capture the fact that the positive sentence is ungrammatical

Conclusions:

We cannot use EXH based on IE for ‘any’;
We cannot use contradiction generating EXH for scalar implicatures;
We need both EXH