# A unified semantics for exceptive-additive *besides*

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# Overview

Introduction

Alternatives for **besides** vs. alternatives for **but**

Embedded occurrences

wh-Questions with **besides**

More on wh-questions

# Parallels between **besides** and exceptive **but**/**except**

Combined with universal quantifiers all of **but**, **except**, and **besides** yield an exceptive interpretation.

(1) **Every student but/except/besides Ann passed.**
$\leadsto$ *Ann is a student* — containment
$\leadsto$ *Every student who is not Ann passed* — quantification
$\leadsto$ *Ann didn't pass* — exception

(2) **No student but/except/besides Ann passed.**
$\leadsto$ *Ann is a student* — containment
$\leadsto$ *No student who is not Ann passed* — quantification
$\leadsto$ *Ann passed* — exception

# Differences between **besides** and exceptive **but**/**except**

Combined with non-universal quantifiers **besides** yields an additive interpretation.

(3) **Some student *but/*except/besides Ann passed.**
⇝ *Ann is a student* containment
⇝ *Some student who is not Ann passed* quantification
⇝ *Ann passed* addition

(4) **At least/more than/one student *but/*except/besides Ann passed.**
⇝ *Ann is a student* containment
⇝ *At least/more than one student who is not Ann passed* quantification
⇝ *Ann passed* addition

(5) **At most/fewer than two students *but/*except/besides Ann passed.**
⇝ *Ann is a student* containment
⇝ *At most/fewer than two students who are not Ann passed* quantification
⇝ *Ann passed* addition

# von Fintel's take on **but**

(6) **Every student but Ann passed.**

**but** makes at least two semantic contributions:

- set subtraction deriving quantification inference:

  $(\{x : x \text{ is a student}\} - \{\text{Ann}\}) \subseteq P$

- leastness naming unique exception and deriving exceptive inference:

  $\{x : x \text{ is a student}\} \not\subseteq P$

(von Fintel 1993)

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- leastness naming unique exception and deriving exceptive inference:

  $\{x : x \text{ is a student}\} \nsubseteq P$

(von Fintel 1993)

# Scope of the exceptive inference I

The exceptive inference of **besides** can take scope independent of set subtraction.

(7) Situation: Ann, Betty, Carl and Dan are the students. John thinks Betty, Carl, and Dan didn't pass. He is not sure about Ann.
**John is certain that no student besides Ann passed.**

a. #'John is certain that no student who is not Ann passed and that Ann passed.' (NS subtraction + exception)
b. 'John is certain that no student who is not Ann passed and is not certain that Ann passed.' (NS subtraction, WS exception)
c. #'For no student who is not Ann is John certain that they passed and he is certain that Ann passed.' (WS subtraction + exception)

The exceptive inference in (7) cannot be assumed to be optional:

(8) Situation: John thinks none of Ann, Betty, Carl, and Dan passed.
#**John is certain that no student besides Ann passed.**

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(8) Situation: John thinks none of Ann, Betty, Carl, and Dan passed.
#**John is certain that no student besides Ann passed.**

# Scope of the exceptive inference II

(9) Situation: Ann works for a bomb disposal unit. When diffusing a bomb with three red buttons and one green one, she is told to not press any red buttons as the bomb will go off. The only harmless button is the green one. So she can press the green button.

a. **Ann is required to press no button besides the green one.**

b. **Ann is required to not press any button besides the green one.**

c. #'Ann is required to not press any red button but to press the green one.' (NS subtraction + exception)

d. 'Ann is required to not press any red button but is allowed to press the green one.' (NS subtraction, WS exception)

e. #'For any red button Ann is not required to press it and she is required to press the green one.' (WS subtraction + exception)

# Decomposition of **but** plus **Exh**

**But** contributes set subtraction.

$[\![\textbf{but}]\!] = \lambda P_{et}.\lambda Q_{et} : P \subseteq Q \ . \ Q - P$

Alternatives to **No student but Ann passed** vary in the position after **but**.

$$Alt \subseteq \left\{ \begin{array}{c} \textbf{no student but Ann passed} \\ \textbf{no student but Bill passed} \\ \textbf{no student but Carl passed} \\ \dots \end{array} \right\}$$

**Exh** contributes exception by negating all the alternatives.

Depending on where it applies the exception inference has varying strength.

$[\![$ **[[ Exh$_{Alt}$ [[ no student [ but Ann$_F$ ]] passed ]]** $]\!]^g$

$$\begin{aligned} = 1 \text{ iff } \{B, C, D\} \cap P = \varnothing \wedge \{A, C, D\} \cap P \neq \varnothing \\ \wedge \{A, B, D\} \cap P \neq \varnothing \\ \wedge \{A, B, C\} \cap P \neq \varnothing \end{aligned}$$

(Gajewski 2008, Hirsch 2016, Crnič 2018, 2021)

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$$\begin{aligned} &[\![\,[\ \textbf{Exh}_{Alt}\ [\![\ \textbf{no student}\ [\ \textbf{but Ann}_F\ ]\!]\ \textbf{passed}\ ]\!]\,]\!]^g \\ &= 1 \text{ iff } \{B, C, D\} \cap P = \varnothing \wedge \{A, C, D\} \cap P \neq \varnothing \\ &\qquad\qquad\qquad\qquad\qquad\;\; \wedge \{A, B, D\} \cap P \neq \varnothing \\ &\qquad\qquad\qquad\qquad\qquad\;\; \wedge \{A, B, C\} \cap P \neq \varnothing \end{aligned}$$

(Gajewski 2008, Hirsch 2016, Crnič 2018, 2021)

# Scope of the additive inference

The additive inference can also take scope independently from set subtraction.

(10) <u>Situation:</u> The requirement for a literature class is that every student read at least one Russian novel other than *War and Peace* from a list of ten books. *War and Peace* is non-mandatory but recommended.
**Ann is required to read at least one book besides *War and Peace*.**

a. #'Ann is required to read at least one book that is not W&P and to read W&P.'

b. 'Ann is required to read at least one book that is not W&P and she is not required to read W&P.'

# Predicted ungrammaticality

(11) ***At least one student but Ann passed.**

Whenever the prejacent is true, there are alternatives that cannot be negated without incurring a contradiction.

$$
\begin{aligned}
&[\![\text{[ Exh}_{Alt}\text{ [[ at least one student [ but Ann}_F\text{ ]] passed ]]}]\!]^g \\
&= 1 \text{ iff } |\{B, C, D\} \cap P| \geq 1 \wedge |\{A, B, D\} \cap P| < 1 \\
&\qquad\qquad\qquad\qquad\qquad\quad \wedge |\{A, B, C\} \cap P| < 1 \\
&\qquad\qquad\qquad\qquad\qquad\quad \wedge |\{A, C, D\} \cap P| < 1 \\
&= 0
\end{aligned}
$$

# The dilemma

Adopting the strategy for **but** also for **besides** accounts for the exceptive inferences and for their varying degrees of strength

At the same time it predicts the cases with additive inferences to be ungrammatical.

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# Different alternatives

We'll assume **besides** is like **but**.

$[\![\textbf{besides}]\!] = \lambda P_{et}.\lambda Q_{et} : P \subseteq Q \ . \ Q - P$

But **Exh** makes use of alternatives to **besides**:

$Alt$(**every student besides$_F$ Ann passed**) =

$$\left\{ \begin{array}{c} \textbf{every student besides Ann passed} \\ \textbf{every student (including Ann) passed} \end{array} \right\}$$

$[\![\textbf{Exh}_{Alt}$ **[ every student besides Ann passed ]**$]\!]$
$= 1$ iff $\{B, C, D\} \subseteq P \wedge \{A, B, C, D\} \nsubseteq P$
$= 1$ iff $\{B, C, D\} \subseteq P \wedge A \notin P$

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# Plural exceptions

To account for sentences with pluralities we assume that these contribute subdomain alternatives:

$$Alt = \left\{ \begin{array}{c} \text{every student besides Ann and Bill passed} \\ \text{every student besides Ann passed} \\ \text{every student besides Bill passed} \\ \text{every student passed} \end{array} \right\}$$

$[\![\text{Exh}_{Alt}$ [ every student besides Ann and Bill was there $]]\!]$

$$\begin{aligned} = 1 \text{ iff } \{C, D\} \subseteq P &\wedge \{B, C, D\} \nsubseteq P \\ &\wedge \{A, C, D\} \nsubseteq P \\ &\wedge \{A, B, C, D\} \nsubseteq P \end{aligned}$$

$= 1 \text{ iff } C \in P \wedge D \in P \wedge A \notin P \wedge B \notin P$

(Bar-Lev 2021)

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$$\begin{aligned} &= 1 \text{ iff } \{C, D\} \subseteq P \wedge \{B, C, D\} \nsubseteq P \\ &\qquad\qquad\qquad\qquad \wedge \{A, C, D\} \nsubseteq P \\ &\qquad\qquad\qquad\qquad \wedge \{A, B, C, D\} \nsubseteq P \\ &= 1 \text{ iff } C \in P \wedge D \in P \wedge A \notin P \wedge B \notin P \end{aligned}$$

(Bar-Lev 2021)

# Exactly *n* plus **besides**

$$Alt = \left\{ \begin{array}{c} \text{exactly one student besides Ann passed} \\ \text{exactly one student passed} \end{array} \right\}$$

$$[\![ \text{Exh}_{Alt} \text{ [ exactly one student besides Ann passed ]} ]\!]$$
$$= 1 \text{ iff } !1x \in \{B, C, D\}[x \in P] \quad \wedge \quad \neg !1x \in \{A, B, C, D\}[x \in P]$$

$$= 1 \text{ iff } \begin{array}{l} (B \in P \wedge C \notin P \wedge D \notin P) \vee \\ (B \notin P \wedge C \in P \wedge D \notin P) \vee \\ (B \notin P \wedge C \notin P \wedge D \in P) \end{array} \wedge \begin{array}{l} [r](A \notin P \wedge B \notin P \wedge C \notin P \wedge D \notin P) \vee \\ (A \in P \wedge B \in P \wedge C \notin P \wedge D \notin P) \vee \\ (A \in P \wedge B \notin P \wedge C \in P \wedge D \notin P) \vee \\ \ldots \\ (A \notin P \wedge B \in P \wedge C \in P \wedge D \notin P) \vee \\ \ldots \\ (A \in P \wedge B \in P \wedge C \in P \wedge D \in P) \vee \\ \ldots \end{array}$$

$$= 1 \text{ iff } \begin{array}{l} (A \in P \wedge B \in P \wedge C \notin P \wedge D \notin P) \vee \\ (A \in P \wedge B \notin P \wedge C \in P \wedge D \notin P) \vee \\ (A \in P \wedge B \notin P \wedge C \notin P \wedge D \in P) \end{array}$$

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$$Alt = \left\{ \begin{array}{c} \text{exactly one student besides Ann passed} \\ \text{exactly one student passed} \end{array} \right\}$$

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$$= 1 \text{ iff } \begin{array}{l} (A \in P \wedge B \in P \wedge C \notin P \wedge D \notin P) \vee \\ (A \in P \wedge B \notin P \wedge C \in P \wedge D \notin P) \vee \\ (A \in P \wedge B \notin P \wedge C \notin P \wedge D \in P) \end{array}$$

# Exactly $n$ plus besides

$$Alt = \left\{ \begin{array}{c} \text{exactly one student besides Ann passed} \\ \text{exactly one student passed} \end{array} \right\}$$

$[\![\text{Exh}_{Alt}$ [ exactly one student besides Ann passed ]$]\!]$

$= 1$ iff $!1x \in \{B, C, D\}[x \in P] \quad \wedge \quad \neg!1x \in \{A, B, C, D\}[x \in P]$

$$= 1 \text{ iff } \begin{array}{l} (B \in P \wedge C \notin P \wedge D \notin P) \vee \\ (B \notin P \wedge C \in P \wedge D \notin P) \vee \\ (B \notin P \wedge C \notin P \wedge D \in P) \end{array} \quad \wedge \quad \begin{array}{l} [r](A \notin P \wedge B \notin P \wedge C \notin P \wedge D \notin P) \vee \\ (A \in P \wedge B \in P \wedge C \notin P \wedge D \notin P) \vee \\ (A \in P \wedge B \notin P \wedge C \in P \wedge D \notin P) \vee \\ \ldots \\ (A \notin P \wedge B \in P \wedge C \in P \wedge D \notin P) \vee \\ \ldots \\ (A \in P \wedge B \in P \wedge C \in P \wedge D \in P) \vee \\ \ldots \end{array}$$

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# Plural additivity

Subdomain alternatives also derive plural additive inferences.

$$Alt = \left\{ \begin{array}{c} \text{exactly one student besides Ann and Bill passed} \\ \text{exactly one student besides Ann passed} \\ \text{exactly one student besides Bill passed} \\ \text{exactly one student passed} \end{array} \right\}$$

$[\![\text{Exh}_{Alt}$ [ exactly one student besides Ann and Bill passed ]$]\!]$

$$= 1 \text{ iff } !1x \in \{C, D\}[x \in P] \wedge \neg !1x \in \{A, B, C, D\}[x \in P]$$
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$$= 1 \text{ iff } (A \in P \wedge B \in P \wedge C \in P \wedge D \notin P) \vee$$
$$(A \in P \wedge B \in P \wedge C \notin P \wedge D \in P)$$

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# Gajewski's alternatives yield a different result

$$Alt = \left\{ \begin{array}{c} \text{exactly one student besides Ann passed} \\ \text{exactly one student besides Betty passed} \\ \text{exactly one student besides Carl passed} \\ \text{exactly one student besides Dan passed} \end{array} \right\}$$

$$\begin{aligned} &[\![\text{Exh}_{Alt} \text{ [ exactly one student besides Ann passed ]}]\!] \\ &= 1 \text{ iff } |\{B, C, D\} \cap P| = 1 \wedge |\{A, C, D\} \cap P| \neq 1 \\ &\qquad\qquad\qquad\qquad\qquad \wedge |\{A, B, D\} \cap P| \neq 1 \\ &\qquad\qquad\qquad\qquad\qquad \wedge |\{A, B, C\} \cap P| \neq 1 \\ &= 0 \end{aligned}$$

# Triviality or vacuity

Gajewski's alternatives accounts for the ungrammaticality with **but**:

(12) *__Exactly one student but Ann passed.__

Assuming obligatory **Exh**, ungrammaticality follows either as:

- a trivial meaning, or
- assuming innocent exclusion, as a violation of the ban on vacuous **Exh**.

(Hirsch 2016)

# At least *n* plus besides

The less complex alternative is weaker than the prejacent.

$$Alt = \left\{ \begin{array}{c} \textbf{at least one student besides Ann passed} \\ \textbf{at least one student passed} \end{array} \right\}$$

No exclusion takes place and no additive inference comes about.

A ban on vacuous quantification blocks this LF.

$[\![\text{Exh}_{Alt}$ [ **at least one student besides Ann passed** ]$]\!]$
$= 1$ iff $\geq 1x \in \{B, C, D\}[x \in P]$

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# Disjunction of exactly-statements?

Assume that **at least *n*** is the disjunction of **exactly *n*** and its alternatives **exactly *n+i*** where ***i*** $>$ ***n***.

The predicted truth-conditions for **At least one student besides Ann passed** would then be as follows:

$$(A \in P \wedge B \in P \wedge C \notin P \wedge D \notin P) \vee (A \in P \wedge B \notin P \wedge C \in P \wedge D \notin P) \vee (A \in P \wedge B \notin P \wedge C \notin P \wedge D \in P)$$

$$\vee$$

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# Decomposing modified numerals

There is a silent **exactly** within **at least** left behind by QR.

**[ at least 1 [ $\boldsymbol{\lambda}_d$ [ exactly-d student passed ]]]**

$[\![\text{exactly}]\!] = \lambda n_d.\lambda f_{et}.\lambda g_{et}.|\{x : f(x) = 1\} \cap \{x : g(x) = 1\}| = n$

$[\![\text{at least}]\!] = \lambda n_d.\lambda f_{dt}.\exists d[d \geq n \wedge f(d) = 1]$

$[\![$ [ $\boldsymbol{\lambda}_n$ [ exactly-d student passed ]]$]\!]^g$
$= \lambda d.|\{x : x \text{ is a student}\} \cap P| = d$

$[\![$[ at least 1 [ $\boldsymbol{\lambda}_d$ [ exactly-d student passed ]]]$]\!]^g$
$= 1 \text{ iff } \exists d[d \geq 3 \wedge |\{x : x \text{ is a student}\} \cap P| = d]$

(Heim 2000a, Hackl 2000, Mayr and Meyer 2014)

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# Embedded **Exh**

**[ at least 1 [ $\boldsymbol{\lambda}_d$ [ Exh$_{Alt}$ [[ exactly-d student besides Ann ] passed ]]]]**

**$[\![$[ Exh$_{Alt}$ [ exactly-d student besides Ann passed ]]$]\!]^g$**
$= 1$ iff $!g(d)x \in \{B, C, D\}[x \in P] \wedge \neg !g(d)x \in \{A, B, C, D\}[x \in P]$

**$[\![$[ $\boldsymbol{\lambda}_d$ [ Exh$_{Alt}$ [ exactly-d student besides Ann passed ]]]$]\!]$**
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# More than *n* plus besides

This straightforwardly extends to **more than n**.

$[\![$**more than**$]\!] = \lambda n_d.\lambda f_{dt}.\exists d[d > n \wedge f(d) = 1]$

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# Downward monotone modified numerals

The predicted truth-conditions do not guarantee the additive inference.

$[\![$**fewer than**$]\!] = \lambda n_d.\lambda f_{dt}.\neg\exists d[d \geq n \wedge f(d) = 1]$

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This issue extends to **at most *n***.

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# At-least implicature

A further **Exh** derives an at-least implicature based on the alternative with **zero.**

$$Alt' = \left\{ \begin{array}{c} \textbf{fewer than two students besides Ann passed} \\ \textbf{fewer than zero students besides Ann passed} \end{array} \right\}$$

$[\![$ Exh$_{Alt'}$ [ fewer than 3 [ $\boldsymbol{\lambda}_d$ [ Exh$_{Alt}$ [[ exactly-d student besides Ann ] passed $]]]]\!]$

$$= 1 \text{ iff } \neg\exists d[d \geq 3 \wedge !dx \in \{B, C, D, E\}[x \in P] \wedge \neg !dx \in \{A, B, C, D, E\}[x \in P]] \wedge$$
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$$= 1 \text{ iff } |\{B, C, D, E\} \cap P| < 3 \wedge A \in P$$

This implicature does not entail that anyone besides Ann passed.

The **zero**-alternative is the only excludable one thereby not interfering with any potential uncertainty implicatures.

The same is possible for **at most *n***.

(Mayr 2013, Mayr and Meyer 2014, Schwarz 2016)

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# Indefinites

Assuming that singular indefinites are, at least on one analysis, parallel to modified numerals, the account for **at least one** straightforwardly extends.

(13) **Some student besides Ann passed.**

The same is true for plural indefinites, if they are treated in parallel to **more than 1**.

(14) **Some students besides Ann passed.**

# Overview

# Negative quantifiers below **require**

(15) Situation: Ann works for a bomb disposal unit. When diffusing a bomb with three red buttons and one green one, she is told to not press any red buttons as the bomb will go off. The only harmless button is the green one. So she can press the green button.
**Ann is required to press no button besides the green one.**
'Ann is required to not press any red button but is allowed to press the green one.' (NS subtraction, WS exception)

$[\![$ [ $\text{Exh}_{Alt}$ [ required [ Ann presses no button besides the green one ]]]$]\!]^g$
$= 1 \text{ iff } \Box(\{R_1, R_2, R_3\} \cap P = \varnothing) \wedge \neg\Box(\{G, R_1, R_2, R_3\} \cap P = \varnothing)$
$= 1 \text{ iff } \Box(\{R_1, R_2, R_3\} \cap P = \varnothing) \wedge \Diamond(\{G, R_1, R_2, R_3\} \cap P \neq \varnothing)$
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# Everything below **required**

(16) Situation: All students have to read *War & Peace*. Being the instructor of the class, A knows that the minimal requirement is that one additional book be read.

Q: **How many books is John required to read for this class on Russian literature?**

A: **He is required to read at least one book besides *War and Peace* . . .**
**#And if he doesn't read *War and Peace*, that's fine too.**

**⟦[ required [ at least 1 [ $\boldsymbol{\lambda}_d$ [ Exh$_{Alt}$ [[ exactly-d student besides Ann ] passe**
$= 1$ iff $\Box(\exists d[d \geq 1 \wedge !dx \in \{B, C, D\}[x \in P] \wedge A \in P])$

(Büring 2008)

# **Exh** above **required** plus QR

(17) Situation: All students have to read *War & Peace*. Being a not very attentive student, A is not sure what minimal requirement regarding further reading obligations is.

Q: **How many books is John required to read for this class on Russian literature?**

A: **He is required to read at least one book besides *War and Peace*, but I am not sure how many exactly.**

$[\![$at least 1 $\lambda_d$ [ Exh$_{Alt}$ [ exactly-d book besides W&P $\lambda_3$ [ required John reads $t_3$ $]\!]$

$= 1$ iff $\exists d[d \geq 1 \wedge |\{B, C, D\} \cap \{x : x$ is a book John must read$\}| = d \wedge$

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# **Exh** above **required** but no QR

(18) Situation: Being a not very attentive student, A is not sure what the minimal reading obligations regarding books that are not *War and Peace* are. A thinks *War and Peace* is allowed reading.

Q: **How many books is John required to read for this class on Russian literature?**

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(i) **And if he doesn't read *War and Peace*, that's fine too.**

(ii) **#But he is not allowed to read *War and Peace*.**

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# **Exactly** below **require**

(19) Situation: Out of one green and three red buttons, John is told to press the green button plus exactly one more. He is not allowed to press any further buttons.
**To save the world John is required to press exactly one button besides the green one.**

$[\![$ **[ required [ Exh$_{Alt}$ [ John presses exactly one button besides the green one ]]]** $]\!]^g$
$= 1$ iff $\Box(!1 \in \{R_1, R_2, R_3\}[x \in P] \wedge G \in P])$

# **Exactly** above **require**

(20) Situation: John must press the green button and in addition press a red button. It's possible that he can press more than one red button.
**To save the world John is required to press exactly one button besides the green one.**

$[\![$ **[ Exh$_{Alt}$ [ exactly one button besides the green one $\boldsymbol{\lambda}_3$ [ required John press t$_3$ ]]]** $]\!]$
$= 1$ iff $\exists d[d = 1 \wedge |\{R_1, R_2, R_3\} \cap \{x : x \text{ John must press } x\}| = d \wedge$
$G \in \{x : x \text{ John must press } x\}]$

# Overview

Introduction

Alternatives for **besides** vs. alternatives for **but**

Embedded occurrences

wh-Questions with **besides**

More on wh-questions

# **Which**/**who** with **besides**

Wh-questions with **besides** give rise to additive inferences.

(21) **Who besides Ann passed?** $\rightsquigarrow$ *Ann passed*

(22) **Which girl(s) besides Ann passed?** $\rightsquigarrow$ *Ann passed*

# A problem

**Exh** standardly applies to propositions.

In the case of wh-questions, one might think that **Exh** applies to the true answer to the question to license **besides**.

True answers are by definition true and cannot be negated, however.

# Karttunen semantics plus **besides**

$[\![?]\!] = \lambda p_{st}.\lambda q_{st}.q = p$

$[\![\text{wh}]\!] = \lambda f_{\langle e,st\rangle}.\lambda g_{\langle e,\langle st,t\rangle\rangle}.\lambda w_s.\lambda p_{st}.\exists x[f(x)(w) = 1 \wedge g(x)(p) = 1]$

$[\![\text{-o}]\!] = \lambda x_e.\lambda w_s.x$ is human in $w$

$$[\![[[\text{ wh [ -o ]] } \boldsymbol{\lambda}_2 \text{ [ ? [ t}_2 \text{ passed ]]]]}]\!]^g(w_o) = \left\{ \begin{array}{c} \text{Ann passed} \\ \text{Betty passed} \\ \text{Carl passed} \\ \text{Dan passed} \end{array} \right\}$$

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(Karttunen 1977, Heim 2000b)

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# Weak answers

The weak answer operator $Ans_1$ in (23) applied to a question denotation in world $w$ returns the weak answer in $w$.

$$[\![\mathbf{Ans_1}]\!] = \lambda w_s . \lambda Q_{\langle s,\langle st,t\rangle\rangle} . \bigcap\{p : p \in Q(w) \wedge p(w) = 1\}$$

When not applied to a world of evaluation, the weak answer denotes a relation between worlds.

$$[\![\mathbf{Ans_1}]\!](w)([\![\textbf{who besides Anna passed}]\!]^g)$$
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# Entailment between weak answers

If Ann and Bill passed in $w_o$, the weak answers differ.

$[\![\textbf{Ans}_1]\!](w_o)([\![\textbf{Who besides Ann passed?}]\!]^g) = \lambda w.\text{B passed in } w$

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If only Bill passed in $w_1$, they do not differ.

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Regardless of $w$, the weak answer to **Who passed** in $w$ entails the weak answer to **Who besides Ann passed?** in $w$.

$f \subseteq g$ where $f, g \in D_{\langle s,st \rangle}$ iff $\forall w : f(w) \subseteq g(w)$.

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# **Exh** with questions (to be generalized)

The world argument of **Ans**$_1$ gets abstracted over.

The resulting propositional concept serves as the argument for **Exh**.

**$\lambda_4$ [[ Exh$_{Alt}$ w$_4$ ] $\lambda_3$ [ [ Ans$_1$ w$_3$ ] [ who besides Ann passed ]]]**

With a propositional concept $P$ as prejacent, **Exh** feeds $P$ a world argument $w$.

Its alternatives $Q$ are also propositional concepts, in our case those from the question without **besides**-phrase.

The result returns the set of worlds $w'$ such that $P(w)(w') = 1$ and asserts that $P(w)$ is different from $Q(w)$ if $P$ does not entail $Q$.

$[\![\mathbf{Exh}_{Alt}]\!] = \lambda w_s.\lambda p_{\langle s,st\rangle}.\lambda w'_s.f(w)(w') = 1 \wedge \forall q \in Alt[p \nsubseteq q \rightarrow p(w) \neq q(w)]$

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# Exhaustification of the weak answer plus **besides**

$$Alt = \left\{ \begin{array}{c} \boldsymbol{\lambda}_3 \text{ [[ Ans}_1 \text{ w}_3 \text{ ] [who besides Ann passed ]]} \\ \boldsymbol{\lambda}_3 \text{ [[ Ans}_1 \text{ w}_3 \text{ ] [who passed ]]} \end{array} \right\}$$

$$[\![\boldsymbol{\lambda}_4 \text{ [[ Exh}_{Alt} \text{ w}_4 \text{ ] } \boldsymbol{\lambda}_3 \text{ [ [ Ans}_1 \text{ w}_3 \text{ ] [ who besides Ann passed ]]]]}]\!]^g$$
$$= \lambda w'. \bigcap\{p : p \in [\![\text{who besides A passed}]\!]^g(g(4)) \wedge p(g(4)) = 1\}(w') = 1 \wedge$$
$$\forall q \in Alt[\lambda w''. \bigcap\{p : p \in [\![\text{who besides A passed}]\!]^g(w'') \wedge p(w'') = 1\} \nsubseteq q \rightarrow$$
$$\bigcap\{p : p \in [\![\text{who besides A passed}]\!]^g(g(4)) \wedge p(g(4)) = 1\} \neq q(g(4))]$$

$g(4)$ must be a world in which the weak answers to **Who besides Ann passed?** and **Who passed?** differ.

Given the entailment relation can only be the case if Anna passed in $g(4)$.

# Generalized **Exh**

The fully general definition of **Exh** applicable to propositions and any function from worlds to ultimately propositions alike looks as follows:

$$[\![\textbf{Exh}_{Alt}]\!] = \lambda f_{\langle s_1,\ldots\langle s_n,t\rangle\rangle}.\lambda w^1_s \ldots \lambda w'^n_s.f(w^1)\ldots(w^n) = 1 \wedge \\ \forall g \in Alt[f \nsubseteq q \rightarrow f(w_1) \neq g(w_1)]$$

The presuppositional requirement that the values of the prejacent and the non-weaker alternatives in $w^1$ differ allows for alternatives to receive # in the propositional case.

Given that we are dealing with questions as well, it might make sense to adopt the presuppositional theory of **Exh**.

(Spector and Sudo 2017, Bassi et al. 2021)

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# Overview

# **Who** plus **but**

(23) *__Who but Ann passed?__

With Gajewski's alternatives, the weak answer to (23) differs from all its alternatives if either

- Ann passed, or
- Ann didn't pass and everyone else did

$$[\![\textbf{Who but Ann passed}]\!]^g(w_o) = \left\{ \begin{array}{c} \text{B passed} \\ \text{C passed} \end{array} \right\}$$

$$[\![\textbf{Who but Bill passed}]\!](w_o) = \left\{ \begin{array}{c} \text{A passed} \\ \text{C passed} \end{array} \right\}$$

$$[\![\textbf{Who but Cathy passed}]\!](w_o) = \left\{ \begin{array}{c} \text{A passed} \\ \text{B passed} \end{array} \right\}$$

# Singular **which**-questions

Singular **which**-questions have a uniqueness requirement that exactly one individual make the predicate in the question nucleus true.

(24) Q: **Which student passed?**
A: #**Ann and Betty.**

(25) **John knows which student passed.**
'Exactly one student passed and John knows who.'

**Ans**$_1$ adds a uniqueness presupposition.

Uniqueness requires that there be a maximal true (mention-some) answer to $Q$, i.e., a true answer entailing all other true answers.

$$[\![\textbf{Ans}_1]\!] = \lambda w_s . \lambda Q_{\langle s, \langle st,t \rangle \rangle} : \exists p [p \in Q(w) \wedge p(w) = 1 \wedge \forall q [q \in Q(w) \wedge q(w) = 1 \rightarrow p \subseteq q]] . \bigcap \{p : p \in Q(w) \wedge p(w) = 1\}$$

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# Licensing **besides** in singular **which**-questions

Exactly one of the propositions in each set can be true.

$$[\![\textbf{Which student besides Ann passed?}]\!]^g(w_o) = \left\{ \begin{array}{c} \text{Betty passed} \\ \text{Carl passed} \\ \text{Dan passed} \end{array} \right\}$$

$$[\![\textbf{Which student passed?}]\!]^g(w_o) = \left\{ \begin{array}{c} \text{Ann passed} \\ \text{Betty passed} \\ \text{Carl passed} \\ \text{Dan passed} \end{array} \right\}$$

Applying *Exh* to the weak exhaustive answer to **Which student besides Ann passed?** requires that the value of the weak exhaustive answer to the alternative **Which student passed?** differ from it.

This is the case if the weak exhaustive answer to the latter receives #, i.e., its uniqueness requirement is not satisfied.

I.e., Ann plus another student passed.

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# Closure under conjunction

The denotations for **who**-questions and non-singular **which**-questions are closed under conjunction.

I.e., there is entailment among the members of the sets.

Then the argument made still goes through.

$$[\![\textbf{Which student besides Ann passed?}]\!]^g(w_o) = \left\{ \begin{array}{c} \text{B passed} \\ \text{C passed} \\ \text{D passed} \\ \text{B+C passed} \\ \ldots \\ \text{B+C+D passed} \end{array} \right\}$$

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# Strong answers

All else being equal, a wh-question asks for the strong exhaustive answer rather than the weak exhaustive answer

The strong answer says that the weak answer is what it is in the world of evaluation.

$$[\![\textsf{Ans}_2]\!] = \lambda w_s.\lambda Q_{\langle s,\langle st,t\rangle\rangle}.\lambda w'_s.\bigcap\{p : p \in Q(w) \wedge p(w) = 1\} = \bigcap\{p : p \in Q(w') \wedge p(w') = 1\}$$

(Groenendijk and Stokhof 1984, Heim 1994)

# A complication: non-equivalence of strong answers

If only Ann and Bill passed $w_o$, the strong answers differ:

$[\![\textbf{Ans}_2]\!](w_o)([\![\textbf{Who besides Ann passed?}]\!]^g)$
$= \lambda w.$B passed in $w$ and C and D did not pass in $w$

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# Strong answers with innocent exclusion and **besides**

The strong exhaustive answer can be derived from the weak exhaustive one via **Exh**.

The alternatives are the potential weak exhaustive answers derived by variation of the world argument of $\mathbf{Ans}_1$.

Assume only Ann and Bill passed in $w_o$.

The weak exhaustive answer to **Who besides Ann passed?** in $w_o$ says that Ann and Bill passed.

For the strong exhaustive answer all alternative potential weak exhaustive answers entailing that Carl and Dan passed are negated.

(Klinedinst and Rothschild 2011)

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