

Introduction

Exceptive and exceptive-additive constructions behave similarly when they occur with universal quantifiers.

- (1) Every girl *besides/but/except* Ann came.
Domain subtraction: Every girl who is not Ann came.
Containment: Ann is a girl.
Negative inference: Ann did not come.

The puzzle: Exceptive construction are known to be incompatible with non-universal quantifiers (Horn 1989). Exceptive-additive constructions can occur in such contexts and they give rise to an additive inference.

- (2) Some girl(s) *besides/*but/*except* Ann came.
Domain subtraction: Some girl who is not Ann came.
Containment: Ann is a girl.
Positive inference: Ann came.
- (3) (Exactly) one girl *besides/*but/*except* Ann came.
Domain subtraction: (Exactly) one girl who is not Ann came.
Containment: Ann is a girl.
Positive inference: Ann came.
- (4) At least/more than one girl *besides/*but/*except* Ann came.
Domain subtraction: At least/more than one girl who is not Ann came.
Containment: Ann is a girl.
Positive inference: Ann came.
- (5) Fewer than/at most two girls *besides* Ann came.
Domain subtraction: Fewer than/at most two girls who are not Ann came.
Containment: Ann is a girl.
Positive inference: Ann came.

Every and besides

We follow von Stechow (1993, 1994) and assume that the core meaning of *besides* is domain subtraction.

- (6) $\llbracket \text{besides} \rrbracket^{g,w} = \lambda P_{\langle e,t \rangle} \lambda Q_{\langle e,t \rangle} : P \subseteq Q$. Q-P
- (7) $\llbracket \text{girl besides Ann} \rrbracket^{g,w} = \{x : x \text{ is girl}\} - \{A\}$; is defined only if Ann is a girl

We built on Gajewski (2008, 2013), Hirsch (2016) and Črnjč (2022) and propose that the exceptive-additive inference is contributed by a separate element Exh.

- (8) $\llbracket \text{IP}_2 \text{ Exh } \llbracket \text{IP}_1 \text{ every } [\text{girl } [\text{besides}_F \text{ Ann}] \text{ came}] \rrbracket \rrbracket$

The alternatives are formed by substitution of *besides* by *including* (see von Stechow 1989, Gajewski 2013 for a similar idea).

- (9) Alt: $\{ \lambda w. \forall x [(x \text{ is a girl in } w \ \& \ x \notin \{A\}) \rightarrow x \text{ came in } w];$
 $\lambda w. \forall x [x \text{ is a girl in } w \rightarrow x \text{ came in } w]$

Exh asserts its prejacent IP_1 and negates the only alternative not entailed by it.

- (10) $\llbracket \text{IP}_2 \rrbracket^{g,w} = \forall x [(x \text{ is a girl in } w \ \& \ x \notin \{A\}) \rightarrow x \text{ came in } w] \ \& \ \neg \forall x [x \text{ is a girl in } w \rightarrow x \text{ came in } w]$

A paraphrase: every girl who is not Ann came, but not every girl overall came. This entails that Ann did not come, thus the negative inference is captured.

Exactly n and besides

This extends to *exactly n* numerals.

- (11) $\llbracket \text{IP}_2 \text{ Exh } \llbracket \text{IP}_1 [\text{exactly one girl } [\text{besides}_F \text{ Ann}] \text{ came}] \rrbracket \rrbracket$
- (12) $\llbracket (11) \rrbracket^{g,w} = 1$ iff $\{x : x \text{ is a girl in } w \ \& \ x \notin \{A\}\} \cap \{y : y \text{ came in } w\} = 1$ & $\{x : x \text{ is a girl in } w\} \cap \{y : y \text{ came in } w\} \neq 1$

A paraphrase: exactly one girl who is not Ann came, but not exactly one girl came overall. This is only possible if Ann came.

The problem with at least n

An attempt to give a similar LF to the *at least n* case in (4) does not lead to a well-formed meaning. This is because the quantifier is upward entailing and both alternatives are entailed by the prejacent.

- (13) $\llbracket \text{IP}_2 \text{ Exh } \llbracket \text{IP}_1 [\text{at least one girl } [\text{besides}_F \text{ Ann}] \text{ came}] \rrbracket \rrbracket$
- (14) Alt: $\{ \lambda w. \exists! x [(x \text{ is a girl} \ \& \ x \notin \{A\}) \ \& \ x \text{ came in } w];$
 $\lambda w. \exists! x [x \text{ is a girl} \ \& \ x \text{ came in } w]$

Exh has nothing to negate here. This LF is ruled out by the non-vacuity constraint (Fox and Katzir 2011, Chierchia 2013).

- (15) NON-VACUITY: Exh[A] is infelicitous if Exh[A] is equivalent to A.

The general proposal for modified numerals and indefinites

Our proposal is build on the idea that modified numerals are quantifiers over degrees and they undergo QR to be interpreted (Heim 2000, Hackl 2000, Mayr and Meyer 2014, Buccola and Spector 2016).

- (16) $\llbracket \text{IP}_2 \text{ at least one } \llbracket \text{IP}_1 \text{ exactly } d_1 \text{ girls came} \rrbracket \rrbracket$

At least one undergoes QR and leaves a trace of type d. There is a silent *exactly* within *at least n* left behind by QR¹.

- (17) $\llbracket \text{exactly} \rrbracket^{g,w} = \lambda n_d. \lambda f_{\langle e,t \rangle} \lambda g_{\langle e,t \rangle} . |\{x : f(x) = 1\} \cap \{x : g(x) = 1\}| = n$
- (18) $\llbracket \text{at least} \rrbracket^{g,w} = \lambda n_d. \lambda f_{\langle e,t \rangle} . \exists d [d \geq n \ \& \ f(d) = 1]$
- (19) $\llbracket \text{IP}_1 \rrbracket^{g,w} = \lambda n. |\{x : x \text{ is a girl in } w\} \cap \{y : y \text{ came in } w\}| = n$
- (20) $\llbracket \text{IP}_2 \rrbracket^{g,w} = 1$ iff $\exists d [d \geq 1 \ \& \ |\{x : x \text{ is a girl in } w\} \cap \{y : y \text{ came in } w\}| = d]$

We propose that Exh is merged below the abstraction over degrees.

- (21) $\llbracket \text{IP}_3 \text{ at least one } \llbracket \text{IP}_2 \text{ exactly } d_1 \text{ girl besides}_F \text{ Ann } \rrbracket \rrbracket$
- (22) $\llbracket \text{IP}_1 \rrbracket^{g,w} = 1$ iff $|\{x : x \text{ is a girl in } w \ \& \ x \notin \{A\}\} \cap \{y : y \text{ came in } w\}| = g(1)$ & $|\{x : x \text{ is a girl in } w\} \cap \{y : y \text{ came in } w\}| \neq g(1)$
- (23) $\llbracket \text{IP}_3 \rrbracket^{g,w} = \exists d [d \geq 1 \ \& \ |\{x : x \text{ is a girl in } w \ \& \ x \notin \{A\}\} \cap \{y : y \text{ came in } w\}| = d \ \& \ |\{x : x \text{ is a girl in } w\} \cap \{y : y \text{ came in } w\}| \neq d]$

This straightforwardly extends to *more than one* numerals.

We propose to treat singular indefinites in (2) like *at least one* and plural indefinites like *more than one*.

¹ (A clarification: this could be implemented by using the standard existential *many* instead of *exactly* that is turned into *exactly* by applying a silent *max* operator (as proposed by Buccola and Spector 2016) or another Exh).

Downward monotonic numerals

Giving (5) a similar LF does not result in the truth conditions that guarantee the additive inference.

- (24) $\llbracket \text{fewer than} \rrbracket = \lambda n_d. \lambda f_{\langle e,t \rangle} . \neg \exists d [d \geq n \ \& \ f(d) = 1]$
- (25) $\llbracket \text{IP}_3 \text{ fewer than two } \llbracket \text{IP}_2 \text{ exactly } d_1 \text{ girl besides}_F \text{ Ann } \rrbracket \rrbracket$
- (26) $\llbracket \text{IP}_3 \rrbracket^{g,w} = \neg \exists d [d \geq 2 \ \& \ |\{x : x \text{ is a girl in } w \ \& \ x \notin \{A\}\} \cap \{y : y \text{ came in } w\}| = d \ \& \ |\{x : x \text{ is a girl in } w\} \cap \{y : y \text{ came in } w\}| \neq d]$

A paraphrase: either fewer than 2 girls who are not Ann came or 2 or more girls who are not Ann came and Ann did not come.

A further Exh derives an at-least implicature based on the alternative with *zero* given in (28). The zero-alternative is the only excludable one thereby not interfering with any potential uncertainty implicatures. Negating it does not entail that anyone other than Ann came.

- (27) $\llbracket \text{IP}_4 \text{ Exh } \llbracket \text{IP}_3 \text{ fewer than two}_F \llbracket \text{IP}_2 \text{ exactly } d_1 \text{ girl besides}_F \text{ Ann } \rrbracket \rrbracket \rrbracket$
- (28) $\lambda w. \neg \exists d [d \geq 0 \ \& \ |\{x : x \text{ is a girl in } w \ \& \ x \notin \{A\}\} \cap \{y : y \text{ came in } w\}| = d \ \& \ |\{x : x \text{ is a girl in } w\} \cap \{y : y \text{ came in } w\}| \neq d]$
- (29) $\llbracket \text{IP}_4 \rrbracket^{g,w} = \neg \exists d [d \geq 2 \ \& \ |\{x : x \text{ is a girl in } w \ \& \ x \notin \{A\}\} \cap \{y : y \text{ came in } w\}| = d \ \& \ |\{x : x \text{ is a girl in } w\} \cap \{y : y \text{ came in } w\}| \neq d] \ \& \ \exists d [d \geq 0 \ \& \ |\{x : x \text{ is a girl in } w \ \& \ x \notin \{A\}\} \cap \{y : y \text{ came in } w\}| = d \ \& \ |\{x : x \text{ is a girl in } w\} \cap \{y : y \text{ came in } w\}| \neq d]$

A paraphrase: Ann came and fewer than two girls who are not Ann came.

The difference with exceptives

The relevant difference between exceptives and exceptive-additives is the way the alternatives are constructed. For exceptives, we adopt Hirsch's (2016) proposal, where the alternatives are formed by substituting the DP following *but* by other DPs of at most equal complexity.

- (30) *Exactly one girl but Ann came.
- (31) $\llbracket \text{IP}_2 \text{ Exh } \llbracket \text{IP}_1 [\text{exactly one girl } [\text{but Ann}_F] \text{ came}] \rrbracket \rrbracket$

Let's assume that the set of girls is as follows: {A, B, C, D}. Then the prejacent of Exh means that exactly one of B, C, D came. The set of alternatives is in (32). We can negate maximally 2 of them. Let's assume that only B came. Then the first two alternatives can be false if A also came. However, in this case the last alternative cannot be negated.

- (32) $\{ \lambda w. |\{A, B, C\} \cap \{y : y \text{ came in } w\}| = 1$
 $\lambda w. |\{A, B, D\} \cap \{y : y \text{ came in } w\}| = 1$
 $\lambda w. |\{A, C, D\} \cap \{y : y \text{ came in } w\}| = 1 \}$

No alternative will be in every maximal set of alternatives that can be negated together with the assertion of the prejacent. Thus none of the alternatives will be innocently excludable. Thus, (30) is ruled out by the non-vacuity constraint.

References: Buccola, B. and B. Spector 2016. Modified numerals and maximality. *Ling and Phil* 39(3), 151–199; Chierchia, G. 2013. *Logic in Grammar: Polarity, Free Choice, and Intervention*; Črnjč, L. 2021. *Exceptives and exhaustification*. *Proceedings of WCCFL 39*; Gajewski, J. 2008. NPI any and connected exceptive phrases. *NALS*(16): 69–110; Gajewski, J. 2013. An analogy between a connected exceptive phrase and polarity items. In *Beyond 'Any' and 'Ever': New Explorations in Negative Polarity Sensitivity*: 183–212; Hackl, M. 2000. *Comparative Quantifiers*. Ph. D. thesis, MIT; Heim, I. 2000. Degree operators and scope. In *Proceedings of SALT X*: 40–64; Hirsch, A. 2016. An unexceptional semantics for expressions of exception. In *UPenn Working Papers in Linguistics*: Vol. 22 (1): 138–148; Mayr, C. and M.-C. Meyer. More than at least. talk at Two Days at least workshop (2014, September); von Stechow, K. 1989. *Exception Phrases*. In *Papers on Quantification* UMass: 1–8; von Stechow, K. 1993. *Exceptive constructions*. *NALS* (1): 123–148; von Stechow, K. 1994. *Restrictions on quantifier domains*: UMass, Amherst Ph.D. thesis.